UNIVERSITY OF MADRAS INSTITUTE OF DISTANCE EDUCATION MSc MATHEMATICS Under Choice Based Credits System (With effect from the academic year 2018-2019)

SCHEME OF EXAMINATION

SEMESTER I	SUBJECTS		MA MA	AX RKS	ſAL
COURSE COMPONENT			INT	EXT	TOT
Core Paper-I	Algebra - I	4	20	80	100
Core Paper-II	Real Analysis - I	4	20	80	100
Core Paper-III	Ordinary Differential Equations	4	20	80	100
Core Paper-IV	Graph Theory	4	20	80	100
Elective Paper-I	Discrete Mathematics	3	20	80	100

SEMESTER II	SUBJECTS		MAX MARKS		IAL
COURSE COMPONENT			INT	EXT	TOT
Core Paper-V	Algebra – II	4	20	80	100
Core Paper-VI	Real Analysis – II	4	20	80	100
Core Paper-VII	Partial Differential Equations	4	20	80	100
Core Paper-VIII	Numerical Analysis	4	20	80	100
Elective Paper- II	Java	3	20	80	100

SEMESTER III	SUBJECTS		MA MA	MAX MARKS	
COURSE COMPONENT			INT	EXT	TOJ
Core Paper-IX	Complex Analysis - I	4	20	80	100
Core Paper-X	Topology	4	20	80	100
Core Paper-XI	Operations Research	4	20	80	100
Core Paper-XII	Probability Theory	4	20	80	100
Elective Paper-III	Number Theory Cryptography	3	20	80	100

SEMESTER IV	SUBJECTS		MA MA	AX RKS	TAL
COURSE COMPONENT			INT	EXT	TOT
Core Paper-XIII	Complex Analysis - II	4	20	80	100
Core Paper-XIV	Differential Geometry	4	20	80	100
Core Paper-XV	Functional Analysis	4	20	80	100
Core Paper-XVI	Mechanics	4	20	80	100
Elective Paper-IV	Mathematical Statistics	3	20	80	100

		CREDITS
Core Paper	16 X 4	64
Elective	4 X 3	12
TOTAL		76

CREDIT DISTRIBUTION

MSc- MATHEMATICS Under Choice Based Credits System (With effect from the academic year 2018-2019) SYLLABUS

SEMESTER -I

Core Paper I- ALGEBRA – I

- UNIT I Group actions on a set, Sylow theorems Applications of Sylow theorems. *Chapter 3: Section 3.6 Chapter 4 – Sections 4.2 and 4.3 from J.B. Fraleigh*
- UNIT II Direct products Finite abelian groups- Modules Chapter 2: Sections 2.13 and 2.14 Chapter 4: Section 4.5 from I.N. Herstein
- UNIT III Linear Transformations Canonical forms Triangular form Nilpotent transformations. *Chapter 6: Sections 6.4 , 6.5* from I.N. Herstein
- UNIT IV Jordan form rational canonical form. *Chapter 6 : Sections 6.6 and 6.7* from I.N. Herstein
- UNIT V Trace and transpose Hermitian, unitary, normal transformations, real quadratic form. *Chapter 6 : Sections 6.8, 6.10 and 6.11 (Omit 6.9)* from I.N. Herstein

Recommended Text :

- 1. J.B. Fraleigh, A first course in Abstract Algebra, 5th edition.
- 2. I.N. Herstein. Topics in Algebra (II Edition) Wiley, 2002.

- 1. M.Artin, Algebra, Prentice Hall of India, 1991.
- 2. P.B.Bhattacharya, S.K.Jain, and S.R.Nagpaul, *Basic Abstract Algebra* (II Edition) Cambridge University Press, 1997. (Indian Edition)
- 3. I.S.Luther and I.B.S.Passi, *Algebra*, Vol. I Groups(1996); Vol. II Rings(1999), Narosa Publishing House, New Delhi
- 4. D.S.Dummit and R.M.Foote, *Abstract Algebra*, 2nd edition, Wiley, 2002.
- 5. N.Jacobson, *Basic Algebra*, Vol. I & II W.H.Freeman (1980); also published by Hindustan Publishing Company, New Delhi.

Core Paper – II REAL ANALYSIS –I

UNIT-I : Functions of bounded variation - Introduction - Properties of monotonic functions - Functions of bounded variation - Total variation - Additive property of total variation - Total variation on

[a, x] as a function of x - Functions of bounded variation expressed as the difference of two increasing functions - Continuous functions of bounded variation.

Chapter -6: Sections 6.1 to 6.8

Infinite Series : Absolute and conditional convergence - Dirichlet's test and Abel's test - Rearrangement of series - Riemann's theorem on conditionally convergent series.

Chapter 8 : Sections 8.8, 8.15, 8.17, 8.18

UNIT-II : The Riemann - Stieltjes Integral - Introduction - Notation - The definition of the Riemann - Stieltjes integral - Linear Properties - Integration by parts- Change of variable in a Riemann - Stieltjes integral - Reduction to a Riemann Integral – Euler's summation formula - Monotonically increasing integrators, Upper and lower integrals - Additive and linearity properties of upper and lower integrals - Riemann's condition - Comparison theorems.

Chapter - 7 : Sections 7.1 to 7.14

UNIT-III : The Riemann-Stieltjes Integral - Integrators of bounded variation-Sufficient conditions for the existence of Riemann-Stieltjes integrals-Necessary conditions for the existence of Riemann-Stieltjes integrals- Mean value theorems for Riemann - Stieltjes integrals - The integrals as a function of the interval - Second fundamental theorem of integral calculus-Change of variable in a Riemann integral-Second Mean Value Theorem for Riemann integral-Riemann-Stieltjes integrals depending on a parameter-Differentiation under the integral sign-Lebesgue criteriaon for the existence of Riemann integrals.

Chapter - 7: 7.15 to 7.26

UNIT-IV : Infinite Series and infinite Products - Double sequences - Double series - Rearrangement theorem for double series - A sufficient condition for equality of iterated series - Multiplication of series - Cesaro summability - Infinite products.

Chapter - 8 Sec, 8.20, 8.21 to 8.26

Power series - Multiplication of power series - The Taylor's series generated by a function -Bernstein's theorem - Abel's limit theorem - Tauber's theorem *Chapter 9 : Sections 9.14 9.15, 9.19, 9.20, 9.22, 9.23* **UNIT-V: Sequences of Functions** - Pointwise convergence of sequences of functions -Examples of sequences of real - valued functions - Definition of uniform convergence -Uniform convergence and continuity - The Cauchy condition for uniform convergence -Uniform convergence of infinite series of functions - Uniform convergence and Riemann -Stieltjes integration – Non-uniform Convergence and Term-by-term Integration - Uniform convergence and differentiation - Sufficient condition for uniform convergence of a series -Mean convergence.

Chapter -9 Sec 9.1 to 9.6, 9.8, 9.9, 9.10, 9.11, 9.13

Recommended Text

Tom M.Apostol : *Mathematical Analysis*, 2nd Edition, Narosa, 1989.

- 1. Bartle, R.G. Real Analysis, John Wiley and Sons Inc., 1976.
- 2. Rudin, W. *Principles of Mathematical Analysis*, 3rd Edition. McGraw Hill Company, New York, 1976.
- 3. Malik, S.C. and Savita Arora. *Mathematical Anslysis*, Wiley Eastern Limited.New Delhi, 1991.
- 4. Sanjay Arora and Bansi Lal, *Introduction to Real Analysis*, Satya Prakashan, New Delhi, 1991.
- 5. Gelbaum, B.R. and J. Olmsted, *Counter Examples in Analysis*, Holden day, San Francisco, 1964.
- **6.** A.L.Gupta and N.R.Gupta, *Principles of Real Analysis*, Pearson Education, (Indian print) 2003.

Core Paper III - Ordinary Differential Equations

UNIT-I : Linear equations with constant coefficients Second order homogeneous equations-Initial value problems-Linear dependence and independence-Wronskian and a formula for Wronskian-Non-homogeneous equation of order two. *Chapter 2: Sections 1 to 6*

UNIT-II: Linear equations with constant coefficients

Homogeneous and non-homogeneous equation of order n –Initial value problems-Annihilator method to solve non-homogeneous equation. *Chapter 2 : Sections 7 to 11.*

UNIT-III: Linear equation with variable coefficients

Initial value problems -Existence and uniqueness theorems – Solutions to solve a non-homogeneous equation – Wronskian and linear dependence – Reduction of the order of a homogeneous equation – Homogeneous equation with analytic coefficients-The Legendre equation.

Chapter : 3 Sections 1 to 8 (omit section 9)

UNIT-IV : Linear equation with regular singular points Second order equations with regular singular points –Exceptional cases – Bessel equation .

Chapter 4 : Sections 3, 4 and 6 to 8 (omit sections 5 and 9)

UNIT-V : Existence and uniqueness of solutions to first order equations: Equation with variable separated – Exact equation – Method of successive approximations – the Lipschitz condition – Convergence of the successive approximations and the existence theorem. Chapter 5 : Sections 1 to 6 (omit Sections 7 to 9)

Recommended Text

E.A.Coddington, An introduction to ordinary differential equations (3rd Printing) Prentice-Hall of India Ltd.,New Delhi, 1987.

- 1. Williams E. Boyce and Richard C. Di Prima, *Elementary differential equations and boundary value problems*, John Wiley and sons, New York, 1967.
- 2. George F Simmons, *Differential equations with applications and historical notes*, Tata McGraw Hill, New Delhi, 1974.
- 3. N.N. Lebedev, *Special functions and their applications*, Prentice Hall of India, New Delhi, 1965.
- 4. W.T.Reid. Ordinary Differential Equations, John Wiley and Sons, New York, 1971
- 5. M.D.Raisinghania, Advanced Differential Equations, S.Chand & Company Ltd. New Delhi 2001
- 6. B.Rai, D.P.Choudhury and H.I. Freedman, *A Course in Ordinary Differential Equations*, Narosa Publishing House, New Delhi, 2002.

Core Paper IV - GRAPH THEORY

Pre-requisite: An elementary course in algebra

UNIT-I : Graphs, subgraphs and Trees : Graphs and simple graphs – Graph Isomorphism – The Incidence and Adjacency Matrices – Subgraphs – Vertex Degrees – Paths and

Connection - Cycles - Trees - Cut Edges ana Bonds - Cut Vertices.

Chapter 1 (Section 1.1 – 1.7)

Chapter 2 (Section 2.1 – 2.3)

UNIT-II : Connectivity, Euler tours and Hamilton Cycles : Connectivity – Blocks –

Euler tours – Hamilton Cycles.

Chapter 3 (Section 3.1 – 3.2)

Chapter 4 (Section 4.1 – 4.2)

UNIT-III : Matchings, Edge Colourings : Matchings – Matchings and Coverings in Bipartite Graphs – Edge Chromatic Number – Vizing's Theorem.

Chapter 5 (Section 5.1 – 5.2)

Chapter 6 (Section 6.1 – 6.2)

UNIT-IV : Independent sets and Cliques, Vertex Colourings : Independent sets – Ramsey's Theorem – Chromatic Number – Brooks' Theorem – Chromatic Polynomials.

Chapter 7 (Section 7.1 – 7.2)

Chapter 8 (Section 8.1 – 8.2, 8.4)

UNIT-V: Planar graphs : Plane and planar Graphs – Dual graphs – Euler's Formula – The Five- Colour Theorem and the Four-Colour Conjecture.

Chapter 9 (Section 9.1 – 9.3, 9.6)

Recommended Text

J.A.Bondy and U.S.R. Murthy, Graph Theory and Applications, Macmillan, London, 1976.

- 1. J.Clark and D.A.Holton, A First look at Graph Theory, Allied Publishers, New Delhi, 1995.
- 2. R. Gould. Graph Theory, Benjamin/Cummings, Menlo Park, 1989.
- 3. A.Gibbons, Algorithmic Graph Theory, Cambridge University Press, Cambridge, 1989.
- 4. R.J.Wilson and J.J.Watkins, *Graphs : An Introductory Approach*, John Wiley and Sons, New York, 1989.
- 5. R.J. Wilson, Introduction to Graph Theory, Pearson Education, 4th Edition, 2004, Indian Print.
- 6. S.A.Choudum, A First Course in Graph Theory, MacMillan India Ltd. 1987.

Elective Paper I - DISCRETE MATHEMATICS

UNIT-I: Lattices: Properties of Lattices: Lattice definitions - Modular and distributive lattice; Boolean algebras: Basic properties - Boolean polynomials, Ideals; Minimal forms of Boolean polynomials.

Chapter 1: § 1 A and B § 2A and B. § 3.

UNIT-II: Applications of Lattices: Switching Circuits: Basic Definitions - Applications Chapter 2: § 1 A and B

UNIT-III: Finite Fields

Chapter 3: § 2

UNIT-IV : Polynomials : Irreducible Polynomials over Finite fields – Factorization of Polynomials

Chapter 3: § 3 and §4.

UNIT-V: Coding Theory : Linear Codes and Cyclic Codes

Chapter 4 § 1 and 2

Recommended Text :Rudolf Lidl and Gunter Pilz, Applied Abstract Algebra, Spinger-Verlag, New York, 1984.

- 1. A.Gill, Applied Algebra for Computer Science, Prentice Hall Inc., New Jersey.
- 2. J.L.Gersting, *Mathematical Structures for Computer Science*(3rd Edn.), Computer Science Press, New York.
- 3. S.Wiitala, Discrete Mathematics- A Unified Approach, McGraw Hill Book Co.

SEMESTER – II

Core Paper V - ALGEBRA – II

- **UNIT I** Extension fields Transcendence of e. *Chapter 5: Section 5.1 and 5.2*
- UNIT II Roots or Polynomials.- More about roots Chapter 5: Sections 5.3 and 5.5
- UNIT III Elements of Galois theory. Chapter 5 : Section 5.6
- UNIT IV Finite fields Wedderburn's theorem on finite division rings Chapter 7: Sections 7.1 and 7.2 (Theorem 7.2.1 only)
- **UNIT V** Solvability by radicals Galois groups over the rationals A theorem of Frobenius.

Chapter 5: Sections 5.7 and 5.8 Chapter 7: Sections 7.3

Recommended Text:

I.N. Herstein. Topics in Algebra (II Edition) Wiley 2002

- 1. M.Artin, Algebra, Prentice Hall of India, 1991.
- 2. P.B.Bhattacharya, S.K.Jain, and S.R.Nagpaul, *Basic Abstract Algebra* (II Edition) Cambridge University Press, 1997. (Indian Edition)
- 3. I.S.Luther and I.B.S.Passi, *Algebra*, Vol. I Groups(1996); Vol. II Rings, (1999) Narosa Publishing House, New Delhi.
- 4. D.S.Dummit and R.M.Foote, Abstract Algebra, 2nd edition, Wiley, 2002.
- 5. N.Jacobson, *Basic Algebra*, Vol. I & II Hindustan Publishing Company, New Delhi.

Core Paper VI REAL ANALYSIS – II

Pre-requisite :Real Analysis-I

UNIT-I : Measure on the Real line - Lebesgue Outer Measure - Measurable sets - Regularity - Measurable Functions - Borel and Lebesgue Measurability

Chapter - 2 Sec 2.1 to 2.5 of de Barra

UNIT-II : Integration of Functions of a Real variable - Integration of Non- negative functions - The General Integral - Riemann and Lebesgue Integrals

Chapter - 3 Sec 3.1,3.2 and 3.4 of de Barra

UNIT-III : Fourier Series and Fourier Integrals - Introduction - Orthogonal system of functions - The theorem on best approximation - The Fourier series of a function relative to an orthonormal system - Properties of Fourier Coefficients - The Riesz-Fischer Thorem - The convergence and representation problems in for trigonometric series - The Riemann - Lebesgue Lemma - The Dirichlet Integrals - An integral representation for the partial sums of Fourier series - Riemann's localization theorem - Sufficient conditions for convergence of a Fourier series at a particular point - Cesaro summability of Fourier series- Consequences of Fejes's theorem - The Weierstrass approximation theorem

Chapter 11 : Sections 11.1 to 11.15 of Apostol

UNIT-IV : Multivariable Differential Calculus - Introduction - The Directional derivative - Directional derivative and continuity - The total derivative - The total derivative expressed in terms of partial derivatives - The matrix of linear function - The Jacobian matrix - The chain rule - Matrix form of chain rule - The mean - value theorem for differentiable functions - A sufficient condition for differentiability - A sufficient condition for equality of mixed partial derivatives - Taylor's theorem for functions of \mathbb{R}^n to \mathbb{R}^1

Chapter 12 : Section 12.1 to 12.14 of Apostol

UNIT-V : Implicit Functions and Extremum Problems : Functions with non-zero Jacobian determinants – The inverse function theorem-The Implicit function theorem-Extrema of real valued functions of severable variables-Extremum problems with side conditions.

Chapter 13 : Sections 13.1 to 13.7 of Apostol

Recommended Text :

1. G. de Barra, *Measure Theory and Integration*, New Age International, 2003 (for Units I and II)

2. Tom M.Apostol : *Mathematical Analysis*, 2nd Edition, Narosa 1989 (for Units III, IV and V)

- 1. Burkill, J.C. The Lebesgue Integral, Cambridge University Press, 1951.
- 2. Munroe, M.E. *Measure and Integration*. Addison-Wesley, Mass. 1971.
- 3. Royden, H.L. Real Analysis, Macmillan Pub. Company, New York, 1988.
- 4. Rudin, W. *Principles of Mathematical Analysis*, McGraw Hill Company, New York, 1979.
- 5. Malik, S.C. and Savita Arora. *Mathematical Analysis*, Wiley Eastern Limited. New Delhi, 1991.

Core Paper VII PARTIAL DIFFERENTIAL EQUATIONS

Pre-requisite: UG level differential equations

UNIT-I : Partial Differential Equations of First Order: Formation and solution of PDE-Integral surfaces – Cauchy Problem order eqn- Orthogonal surfaces – First order non-linear – Characteristics – Csmpatible system – Charpit method. **Fundamentals:** Classification and canonical forms of PDE.

Chapter 0: 0.4 to 0.11 (omit .1,0.2.0.3 and 0.11.1) and Chapter 1: 1.1 to 1.5

UNIT-II : Elliptic Differential Equations: Derivation of Laplace and Poisson equation – BVP – Separation of Variables – Dirichlet's Problem and Newmann Problem for a rectangle – Interior and Exterior Dirichlets's problems for a circle – Interior Newmann problem for a circle – Solution of Laplace equation in Cylindrical and spherical coordinates – Examples. **Chapter 2: 2.1, 2 2 , 2.5 to 2.13 (omit 2.3 and 2.4)**

UNIT-III : Parabolic Differential Equations: Formation and solution of Diffusion equation – Dirac-Delta function – Separation of variables method – Solution of Diffusion Equation in Cylindrical and spherical coordinates Examples.

Chapter 3: 3.1 to 3.7 and 3.9 (omit 3.8)

UNIT-IV : **Hyperbolic Differential Equations:** Formation and solution of one-dimensional wave equation – canocical reduction – IVP- d'Alembert's solution – Vibrating string – Forced Vibration – IVP and BVP for two-dimensional wave equation – Periodic solution of one-dimensional wave equation in cylindrical and spherical coordinate systems – vibration of circular membrane – Uniqueness of the solution for the wave equation – Duhamel's Principle – Examples

Chapter 4: 4.1 to 4.12(omit 4.13)

UNIT-V: Green's Function: Green's function for laplace Equation – methods of Images – Eigen function Method – Green's function for the wave and Diffusion equations. **Laplace Transform method:** Solution of Diffusion and Wave equation by Laplace Transform. **Fourier Transform Method:** Finite Fourier sine and cosine franforms – solutions of Diffusion, Wave and Lpalce equations by Fourier Transform Method.

Chapter 5: 5.1 to 5.6 Chapter 6: 6.13.1 and 6.13.2 only (omit (6.14) Chapter 7: 7.10 to 7.13 (omit 7.14)

Recommended Text: S, Sankar Rao, *Introduction to Partial Differential Equations*, 2nd Edition, Prentice Hall of India, New Delhi. 2005

- 1. R.C.McOwen, *Partial Differential Equations*, 2nd Edn. Pearson Eduction, New Delhi, 2005.
- 2. I.N.Sneddon, *Elements of Partial Differential Equations*, McGraw Hill, New Delhi, 1983.
- 3. R. Dennemeyer, *Introduction to Partial Differential Equations and Boundary Value Problems*, McGraw Hill, New York, 1968.
- 4. M.D.Raisinghania, *Advanced Differential Equations*, S.Chand & Company Ltd., New Delhi, 2001.

Core Paper VIII - Numerical Analysis

Pre-requisite :Numerical Analysis

UNIT-I :Non-Linear Equations - Introduction –Bisection Method-Regula-falsi Method-Newton-Raphson Method-Secant Method

UNIT-II : Systems of linear Equations - Introduction –Gauss Elimination-Gauss-Seidal Method

UNIT-III :Interpolation -Introduction - Lagrange's Interpolation Formula.

UNIT-IV :Numerical Differentiation- Differentiation using limits-Differentiation using Extrapolation.

UNIT-V: Numerical Integration - Introduction -Composite Tapezoidal Rule – Composite Simpson 1/3 Rule.

UNIT-VI : Numerical Solution to Differential Equations -Introduction- Euler's Method-Taylor's Method of order 4 -Runge-Kutta Method of order 4 -Milene- Simpson Method.

Recommended Text :Richard L. Burden J.D. Faires Thomson Brook Cole Numerical Analysis

Reference Books

- **1.**Kendall Atkinson, An introduction to numerical analysis, Second Edition, WILEY Publications
- **2.**S.S..Sastry, Introductory Methods of Numerical Analysis ,Fifth Edition , Prentice Hall of India,
- **3.**JohH. Mathes, Numerical Methods for Mathematics, Science and Engineering (2ndEdn.), Prentice Hall, New Delhi, 2000.

4.D.Kincaid & W.Cheney, Numerical Analysis (3rd Ed. Books/Cole)

Elective Paper II JAVA PROGRAMMING

Pre-requisite :Knowledge in Programming in C / C++				
UNIT-I: Java Tokens – Java statements – Constants – Variables – Data types				
Chapters 3 and 4				
UNIT-II : Operators – Expressions – Decision making and Branching.				
Chapters 5,6 and 7				
UNIT-III :				
Classes – Objects – Methods – Arrays – Strings – Vectors – Multiple Inheritance				
Chapters 8, 9 and 10				
UNIT-IV : Multithreaded Programming – Managing errors and Exceptions				
Chapters 12 and 13				
UNIT-V : Applet Programming				
Chapter 14				
Recommended Text : E. Balagurusamy, <i>Programming with Java – A primer</i> , Tata McGraw				
Hill Publishing Company Limited, New Delhi, 1998.				
Reference Books:				
1. Mitchell Waite and Robert Lafore, Data Structures and				
Algorithms in Java, Techmedia (Indian Edition), New Delhi, 1999				

2. Adam Drozdek, *Data Structures and Algorithms in Java*, (Brown/Cole), Vikas Publishing House, New Delhi, 2001.

SEMESTER III

Core Paper IX - COMPLEX ANALYSIS - I

UNIT I - Cauchy's Integral Formula: The Index of a point with respect to a closed curve - The Integral formula - Higher derivatives.
 Local Properties of Analytical Functions : Removable Singularities-Taylors's Theorem-Zeros and poles-The local Mapping - The Maximum Principle . Chapter 4 : Section 2 : 2.1 to 2.3, Section 3 : 3.1 to 3.4

UNIT II - The general form of Cauchy's Theorem : Chains and cycles- Simple Connectivity -Homology - The General statement of Cauchy's Theorem - Proof of Cauchy's theorem – Locally exact differentials-Multilply connected regions – Residue theorem - The argument principle. Chapter 4 : Section 4 : 4.1 to 4.7, Section 5: 5.1 and 5.2

UNIT III - Evaluation of Definite Integrals and Harmonic Functions: Evaluation of definite integrals - Definition of Harmonic functions and basic properties - Mean value property - Poisson formula. Chapter 4 : Section 5 : 5.3, Section 6 : 6.1 to 6.3

- UNIT IV Harmonic Functions and Power Series Expansions: Schwarz theorem - The reflection principle - Weierstrass theorem - Taylor Series - Laurent series . Chapter 4 : Sections 6.4 and 6.5 Chapter 5 : Sections 1.1 to 1.3
- UNIT V Partial Fractions and Entire Functions: Partial fractions Infinite products Canonical products - Gamma Function - Jensen's formula Chapter 5 : Sections 2.1 to 2.4, Section 3.1

Recommended Text :

Lars V. Ahlfors, Complex Analysis, (3rd edition) McGraw Hill Co., New York, 1979

- 1. H.A. Priestly, Introduction to Complex Analysis, Clarendon Press, Oxford, 2003.
- 2. J.B.Conway, *Functions of one complex variable*, Springer International Edition, 2003
- 3. T.W Gamelin, Complex Analysis, Springer International Edition, 2004.
- 4. D.Sarason, Notes on complex function Theory, Hindustan Book Agency, 1998

Core Paper X – TOPOLOGY

- **Unit I** Topological spaces, Basis for a topology, Product topology on X x Y, Subspace topology, Closed sets and Limit points, Continuous functions. *Chapter 2 - Sections 12, 13, 15, 16, 17, 18.*
- **Unit II -** Connected spaces, Connected subspaces of the real line, Components and Local connectedness, Compact spaces, Compact subspaces of the real line. *Chapter 3 Sections 23, 24, 25, 26, 27.*
- Unit III Countability axioms, Separation axioms, Normal spaces, Urysohn Lemma, Urysohn metrization theorem, Tietze extension theorem. *Chapter 4 - Sections 30, 31, 32, 33, 34, 35*.
- Unit IV Product topology, Tychonoff theorem. Chapter 2 - Sections 19. Chapter 5 - Section 37.
- **Unit V -** Homotopy of paths, Fundamental group. *Chapter 9 - Sections 51, 52.*
- Recommended Text : James R. Munkres "Topology" (Second edition) PHI, 2015.

- 1. T.W. Gamelin and R.E. Greene, Introduction to Topology, The Saunders Series, 1983.
- 2. G.F. Simmons, Introduction to Topology and Modern Analysis, Mcgraw-Hill
- 3. J. Dugundji, *Topology*, Prentice Hall of India.
- 4. J.L. Kelly, General Topology, Springer.
- 5. S. Willard, General Topology, Addison-Wesley.

Core Paper XI – OPERATIONS RESEARCH

Pre-requisite : UG Level Operations Research

UNIT-I : Decision Theory : Steps in Decision theory Approach – Types of Decision-Making Environments – Decision Making Under Uncertainty – Decision Making under Risk – Posterior Probabilities and Bayesian Analysis – Decision Tree Analysis – Decision Making with Utilities.

Chapter 10 : Sec. 10.1 to 10.8

UNIT-II : Network Models : Scope of Network Applications – Network Definition – Minimal spanning true Algorithm – Shortest Route problem – Maximum flow model – Minimum cost capacitated flow problem - Network representation – Linear Programming formulation – Capacitated Network simplex Algorithm.

Chapter 6 : Sections 6.1 to 6.6

H.A.Taha : Operations Research

UNIT-III : Deterministic Inventory Control Models: Meaning of Inventory Control – Functional Classification – Advantage of Carrying Inventory – Features of Inventory System – Inventory Model building – Deterministic Inventory Models with no shortage – Deterministic Inventory with Shortages

Probabilistic Inventory Control Models:

Single Period Probabilistic Models without Setup cost – Single Period Probabilities Model with Setup cost.

Chapter 13: Sec. 13.1 to 13.8

Chapter 14: Sec. 14.1 to 14.3

UNIT-IV : Queueing Theory : Essential Features of Queueing System – Operating Characteristic of Queueing System – Probabilistic Distribution in Queueing Systems – Classification of Queueing Models – Solution of Queueing Models – Probability Distribution of Arrivals and Departures – Erlangian Service times Distribution with k-Phases.

Chapter 15 : Sec. 15.1 to 15.8

UNIT-V : Replacement and Maintenance Models: Failure Mechanism of items – Replacement of Items that deteriorate with Time – Replacement of items that fail completely – other Replacement Problems.

Chapter 16: Sec. 16.1 to 16.5

Recommended Text :

- 1. For Unit 2 : H.A. Taha, Operations Research, 6th edition, Prentice Hall of India
- 2. For all other Units: J.K.Sharma, *Operations Research*, MacMillan India, New Delhi, 2001.

Reference Books

1. F.S. Hiller and J.Lieberman -,*Introduction to Operations Research* (7th Edition), Tata McGraw Hill Publishing Company, New Delhui, 2001.

2. Beightler. C, D.Phillips, B. Wilde *Foundations of Optimization* (2nd Edition) Prentice Hall Pvt Ltd., New York, 1979

3. Bazaraa, M.S; J.J.Jarvis, H.D.Sharall *,Linear Programming and Network flow,* John Wiley and sons, New York 1990.

4. Gross, D and C.M.Harris, *Fundamentals of Queueing Theory*, (3rd Edition), Wiley and Sons, New York, 1998.

Core Paper XII – PROBABILITY THEORY

Pre-requisite: UG level calculus and real analysis

UNIT-I : **Random Events and Random Variables:** Random events – Probability axioms – Combinatorial formulae – conditional probability – Bayes Theorem – Independent events – Random Variables – Distribution Function – Joint Distribution – Marginal Distribution – Conditional Distribution – Independent random variables – Functions of random variables.

Chapter 1: Sections 1.1 to 1.7

Chapter 2 : Sections 2.1 to 2.9

UNIT-II : Parameters of the Distribution : Expectation- Moments – The Chebyshev Inequality – Absolute moments – Order parameters – Moments of random vectors – Regression of the first and second types.

Chapter 3 : Sections 3.1 to 3.8

UNIT-III: Characteristic functions : Properties of characteristic functions – Characteristic functions and moments – semi0invariants – characteristic function of the sum of the independent random variables – Determination of distribution function by the Characteristic function – Characteristic function of multidimensional random vectors – Probability generating functions.

Chapter 4 : Sections 4.1 to 4.7

UNIT-IV : Some Probability distributions: One point , two point , Binomial – Polya – Hypergeometric – Poisson (discrete) distributions – Uniform – normal gamma – Beta – Cauchy and Laplace (continuous) distributions.

Chapter 5 : Section 5.1 to 5.10 (Omit Section 5.11)

UNIT-V: Limit Theorems : Stochastic convergence – Bernaulli law of large numbers – Convergence of sequence of distribution functions – Levy-Cramer Theorems – de Moivre-Laplace Theorem – Poisson, Chebyshev, Khintchine Weak law of large numbers – Lindberg Theorem – Lapunov Theroem – Borel-Cantelli Lemma - Kolmogorov Inequality and Kolmogorov Strong Law of large numbers.

Chapter 6 : Sections 6.1 to 6.4, 6.6 to 6.9 , 6.11 and 6.12. (Omit Sections 6.5, 6.10,6.13 to 6.15)

Recommended Text :M. Fisz, *Probability Theory and Mathematical Statistics*, John Wiley and Sons, New York, 1963.

Reference Books

1. R.B. Ash, Real Analysis and Probability, Academic Press, New York, 1972

2. K.L.Chung, A course in Probability, Academic Press, New York, 1974.

4. R.Durrett, *Probability : Theory and Examples*, (2nd Edition) Duxbury Press, New York, 1996.

5. V.K.Rohatgi An Introduction to Probability Theory and Mathematical Statistics, Wiley Eastern Ltd., New Delhi, 1988(3rd Print).

6. S.I.Resnick, A Probability Path, Birhauser, Berlin, 1999.

7. B.R.Bhat, *Modern Probability Theory* (3rd Edition), New Age International (P)Ltd, New Delhi, 1999

Elective Paper III – NUMBER THEORY AND CRYPTOGRAPHY

Pre-requisite: Elementary number theory and calculus

UNIT-I:

Elementary Number Theory: Time Estimates for doing arithmetic – divisibility and Euclidean algorithm – Congruences – Application to factoring. (Chapter 1)

UNIT-II:

Introduction to Classical Crypto systems – Some simple crypto systems – Enciphering matrices DES (Chapter 3)

UNIT-III :

Finite Fields, Quadratic Residues and Reciprocity (Chapter 2)

UNIT-IV:

Public Key Cryptography (Chapter 4)

UNIT-V:

Primality, Factoring, Elliptic curves and Elliptic curve crypto systems (Chapter 5, sections 1,2,3 &5 (omit section 4), Chapter 6, sections 1& 2 only)

Recommended Text: Neal Koblitz, *A Course in Number Theory and Cryptography*, Springer-Verlag, New York, 1987

Reference Books :

1.I. Niven and H.S.Zuckermann, *An Introduction to Theory of Numbers* (Edn. 3), Wiley Eastern Ltd., New Delhi, 1976

- 2. David M.Burton, *Elementary Number Theory*, Brown Publishers, Iowa, 1989
- 3. K.Ireland and M.Rosen, A Classical Introduction to Modern Number Theory, Springer Verlag, 1972

4. N.Koblitz, Algebraic Aspects of Cryptography, Springer 1998

SEMESTER IV

Core Paper - XIII COMPLEX ANALYSIS - II

Pre-requisite: Complex Analysis-I and Real Analysis

UNIT-I: Riemann Zeta Function and Normal Famalies:

Product development – Extension of $\zeta(s)$ to the whole plane – The zeros of zeta function – Equicontinuity – Normality and compactness – Arzela's theorem – Families of analytic functions – The Classcial Definition

Chapter 5 : Sections 4.1 to 4.4, Sections 5.1 to 5.5

UNIT-II : Riemann mapping Theorem : Statement and Proof – Boundary Behaviour – Use of the Reflection Principle.

Conformal mappings of polygons : Behaviour at an angle

Schwarz-Christoffel formula – Mapping of a rectangle.

Harmonic Functions : Functions with mean value property –

Harnack's principle.

Chapter 6 : Sections 1.1 to 1.3 (Omit Section 1.4)

Sections 2.1 to 2.3 (Omit section 2.4), Section 3.1 and 3.2

UNIT-III : Elliptic functions : Simply periodic functions – Doubly periodic functions **Chapter 7 : Sections 1.1 to 1.3, Sections 2.1 to 2.4**

UNIT-IV : Weierstrass Theory : The Weierstrass \wp -function – The functions $\zeta(s)$ and

 $\sigma(s)$ – The differential equation – The modular equation $\lambda(\tau)$ – The Conformal mapping by $\lambda(\tau)$.

Chapter 7 : Sections 3.1 to 3.5

UNIT-V: Analytic Continuation : The Weiesrtrass Theory – Germs and Sheaves – Sections and Riemann surfaces – Analytic continuation along Arcs – Homotopic curves – The Monodromy Theorem – Branch points.

Chapter 8 : Sections 1.1 to 1.7

Recommended Text: Lars V. Ahlfors, *Complex Analysis*, (3rd Edition) McGraw Hill Book Company, New York, 1979.

Reference Books

1.H.A. Priestly, Introduction to Complex Analysis, Clarendon Press, Oxford, 2003.

2.J.B.Conway, Functions of one complex variable, Springer International Edition, 2003

3.T.W Gamelin, Complex Analysis, Springer International Edition, 2004.

4.D.Sarason, Notes on Complex function Theory, Hindustan Book Agency, 1998

Core Paper XIV - DIFFERENTIAL GEOMETRY

Unit I - Curves in the plane and in space :

Curves, parametrisation, arc length, level curves, curvature, plane and space curves.

Chapters 1 and 2.

Unit II - Surfaces in space :

Surface patches, smooth surfaces, tangents, normals, orientability, examples of surfaces, lengths of curves on surfaces, the first fundamental form, isometries, surface area.

Chapter 4 - 4.1, 4.2, 4.3, 4.4, 4.7 and Chapter 5 - 5.1, 5.2, 5.4

Unit III - Curvature of surfaces:

The second fundamental form, Curvature of curves on a surface, normal, principal, Gaussian and mean curvatures, Gauss map. *Chapter 6 - 6.1, 6.2, 6.3 and Chapter 7 - 7.1, 7.5,7.6*

Unit IV - Geodesics :

Geodesics, geodesic equations, geodesics as shortest paths, geodesic coordinates. *Chapter 8 - 8.1, 8.2, 8.4, 8.5*

Unit V - Theorema Egregium of Gauss :

Theorema Egregium, isometries of surfaces, Codazzi-Mainardi equations, compact surfaces of constant Gaussian curvature. *Chapter 10*

Recommended Text:

A. Pressley, *Elementary Differential Geometry*, Springer-Indian Edition, 2004.

<u>Reference Books</u> :

- 1. J.A. Thorpe, *Elementary Topics in Differential Geometry*, Springer-Indian edition.
- 2. E.D. Bloch, A First Course in Geometric Topology and Differential Geometry, Birkhauser, 1997.
- 3. M.P. do Carmo, Differential Geometry of Curves and Surfaces, Prentice-Hall, 1976.

Core Paper XV - FUNCTIONAL ANALYSIS

- Unit I Normed spaces, Continuity of linear maps, Hahn-Banach Theorems, Banach Spaces. *Chapters II (omit sections 6.8, 7.11, 7.12, 8.4)*
- **Unit II -** Uniform boundedness principle, Closed Graph and Open Mapping theorems, Bounded Inverse Theorem, Spectrum of a bounded operator. *Chapter III (omit sections 9.4 to 9.7, 11.2, 11.4, 11.5, 12.6, 12.7)*
- Unit III Duals and Transposes, Weak and weak *convergence, Reflexivity Chapter IV (omit sections 13.7, 13.8, 14, 15.5 to 15.7, 16.5 to 16.9)
- Unit IV Inner Product Spaces, Orthonormal sets, Best approximation, Projection and Riesz Representation theorems.
 Chapter VI (omit sections 23.2, 23.4, 23.6, 24.7, 24.8)
- Unit V Bounded operators and adjoints, Normal, unitary and self adjoint Operators, Spectrum and Numerical range, Compact selfadjoint operators *Chapter VII (omit sections 26.4, 26.5 26.6, 27.4 to 27.7, 28.7, 28.8)*

<u>Recommended Text</u> :

B.V. Limaye, Functional Analysis, New Age International, 1996.

- 1. W.Rudin Functional Analysis, Tata McGraw-Hill Publishing Company, New Delhi, 1973
- 2. G.Bachman & L.Narici, Functional Analysis Academic Press, New York, 1966.
- 3. C. Goffman and G.Pedrick, First course in Functional Analysis, Prentice Hall of India, New Delhi, 1987
- 4. E. Kreyszig, Introductory Functional Analysis with Applications, John wiley & Sons, New York., 1978.
- 5. M.Thamban Nair, Functional Analysis. A First Course, Prentice Hall of India, New Delhi, 2002

Core Paper XVI – MECHANICS

Pre-requisite: Calculus and Differential equations.

UNIT-I : *Mechanical Systems* : The Mechanical system- Generalised coordinates – Constraints - Virtual work - Energy and Momentum Chapter 1 : Sections 1.1 to 1.5

UNIT-II : *Lagrange's Equations*: Derivation of Lagrange's equations- Examples-Integrals of motion. **Chapter 2 : Sections 2.1 to 2.3 (Omit Section 2.4)**

UNIT-III : *Hamilton's Equations* : *Hamilton's Principle - Hamilton's Equation - Other* variational principles.

Chapter 4 : Sections 4.1 to 4.3 (Omit section 4.4)

UNIT – IV : *Hamilton-Jacobi Theory* : *Hamilton Principle function – Hamilton-Jacobi Equation - Separability* Chapter 5 : Sections 5.1 to 5.3

UNIT-V : *Canonical Transformation : Differential forms and generating functions – Special Transformations – Lagrange and Poisson brackets.*

Chapter 6 : Sections 6.1, 6.2 and 6.3 (omit sections 6.4, 6.5 and 6.6)

Recommended Text: D. Greenwood, *Classical Dynamics*, Prentice Hall of India, New Delhi, 1985.

1. H. Goldstein, *Classical Mechanics*, (2nd Edition) Narosa Publishing House, New Delhi.

2. N.C.Rane and P.S.C.Joag, *Classical Mechanics*, Tata McGraw Hill, 1991.

3. J.L.Synge and B.A.Griffth, *Principles of Mechanics* (3rd Edition) McGraw Hill Book Co., New York, 1970.

Elective Paper IV – MATHEMATICAL STATISTICS

Pre-requisite :Basic Probability Theory

UNIT-I : Sample Moments and their Functions: Notion of a

sample and a statistic – Distribution functions of X, S^2 and

 $(~X~,~S^2~)$ - χ^2 distribution – Student t-distribution – Fisher's Z-distribution – Snedecor's F-distribution – Distribution of sample mean from non-normal populations

Chapter 9 : Sections 9.1 to 9.8

UNIT-II : Significance Test : Concept of a statistical test – Parametric tests for small samples and large samples - χ^2 test – Kolmogorov Theorem – Smirnov Theorem – Tests of Kolmogorov and Smirnov type – The Wald-Wolfovitz and Wilcoxon-Mann-Whitney tests – Independence Tests by contingency tables.

Chapter 10 : Sections 10.11

Chapter 11 : 12.1 to 12.7.

UNIT-III : Estimation : Preliminary notion – Consistency estimation – Unbiased estimates – Sufficiency – Efficiency – Asymptotically most efficient estimates – methods of finding estimates – confidence Interval.

Chapter 13 : Sections 13.1 to 13.8 (Omit Section 13.9)

UNIT-IV: Analysis of Variance : One way classification and two-way classification.

Hypotheses Testing: Poser functions – OC function- Most Powerful test – Uniformly most powerful test – unbiased test.

Chapter 15 : Sections 15.1 and 15.2 (Omit Section 15.3)

Chapter 16 : Sections 16.1 to 16.5 (Omit Section 16.6 and 16.7)

UNIT-V : Sequential Analysis : SPRT – Auxiliary Theorem – Wald's fundamental identity – OC function and SPRT – E(n) and Determination of A and B – Testing a hypothesis concerning p on 0-1 distribution and m in Normal distribution.

Chapter 17 : Sections 17.1 to 17.9 (Omit Section 17.10)

Recommended Text: M. Fisz, *Probability Theory and Mathematical Statistics*, John Wiley and sons, New Your, 1963.

- 1. **E.J.Dudewicz and S.N.Mishra**, *Modern Mathematical Statistics*, John Wiley and Sons, New York, 1988.
- 2. V.K.Rohatgi An Introduction to Probability Theory and Mathematical Statistics, Wiley Eastern New Delhi, 1988(3rd Edn)
- 3. **G.G.Roussas**, *A First Course in Mathematical Statistics*, Addison Wesley Publishing Company, 1973
- 4. **B.L.Van der Waerden**, *Mathematical Statistics*, G.Allen & Unwin Ltd., London, 1968.